**Gravitational Fluid**

Let’s look at the physics of a large fluid, mutually interacting via the gravitational force. This will be particularly relevant in GR. We’ll start with the Navier-Stokes equation:

Applying this equation eliminates the ρ derivatives in N2L, giving us:



(renaming Pth as simply P) We’ll actually presume a static fluid. This simplifies our equation to:



The force (density) is **f** = ρ**g**, where **g** is the local gravitational field. And as we saw in a previous file, the gravitational field satisfies:



In terms of the gravitational potential, g = -∇Φ, we can write,



This is called the Newton-Poisson equation. And in terms of this potential, the NS equation becomes,



So our equations are:



These two equations determine the behavior of our static fluid. We’d like to get an equation relating ρ and P directly, specifically, for a spherically symmetric mass distribution (like a star). Then we have:



which simplifies somewhat to:



Let’s solve for u = ∂Φ/∂r.



Therefore,



We can define that integral as m(r) – the mass up to radius r. And then since ρ∂Φ/∂r = -∂P/∂r, we have:

